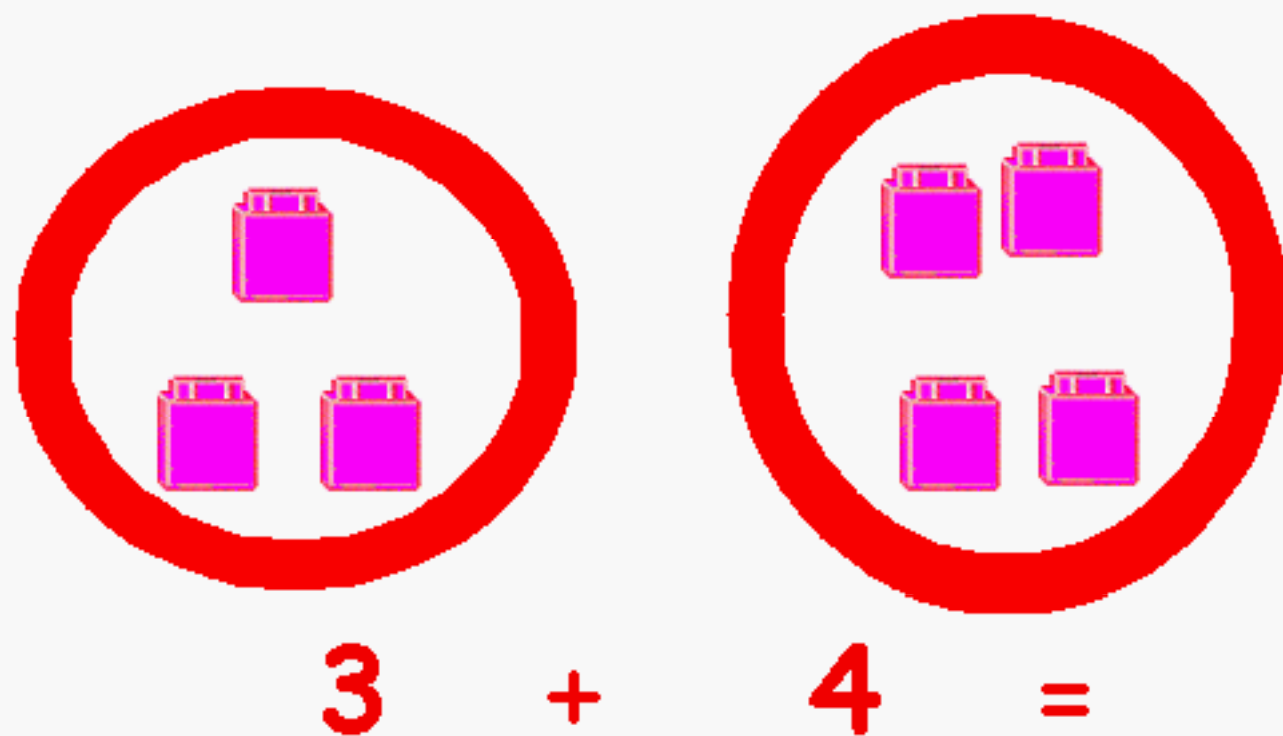


Calculations - Addition

Year 1

Children start by using real objects, combining the 2 groups and counting the total number of objects.



Year 1

Children are taught to add numbers to 10 using their fingers



3

+



5

Year 1

3

Addition to 20 and beyond

Children are encouraged to put the largest number in their head and then count on using their fingers.



17

+



8



=

Year 1 Adding numbers up to 20

4

Children are then taught to find the largest number on a number strip and then counting on the smaller number.



6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20

$$9 + 5 =$$

Year 1 The hundred square

When adding a 1 digit number to a 2 digit number the children find the largest number on a 100 square and the count on the smaller number.

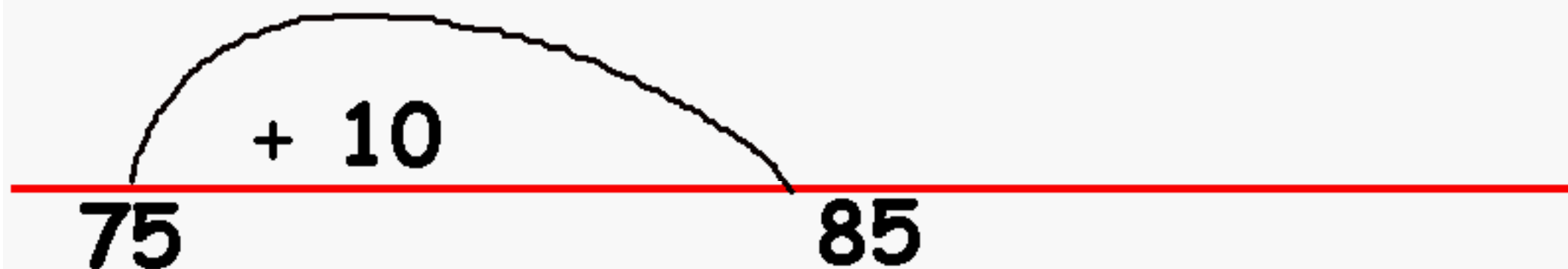
$$53 + 6 =$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Year 2 The empty number line

The children are taught to add 10 using an empty number line. They start by putting the largest number on the empty number line and then drawing on the jump of 10.

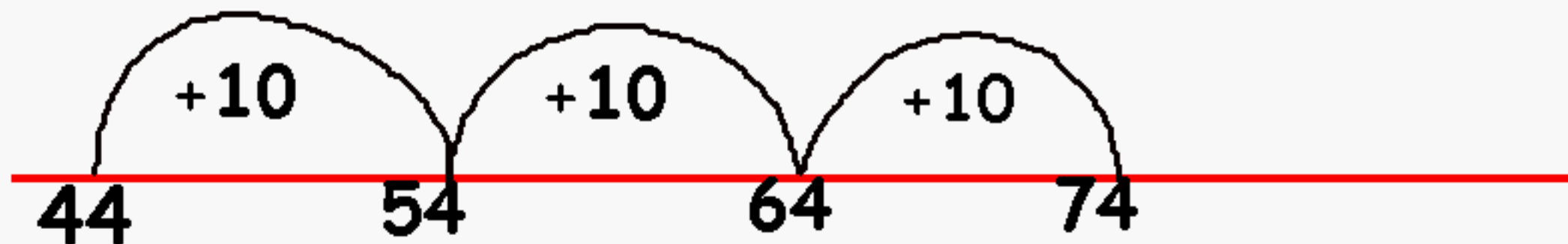
$$75 + 10 =$$



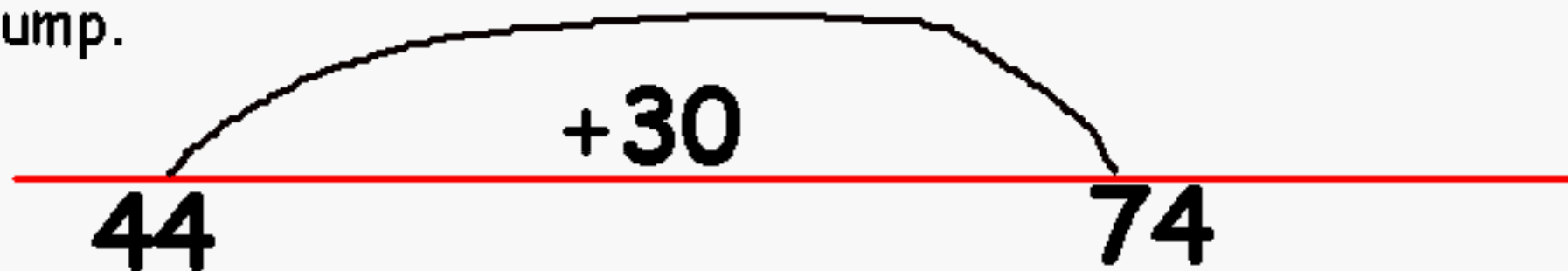
Year 2 The empty number line

Children are taught to put the largest number on the number line and then draw on the multiples of 10.

$$44 + 30 =$$



Some children who are confident to add multiples of 10 in their head will add all the multiples of 10 in one jump.

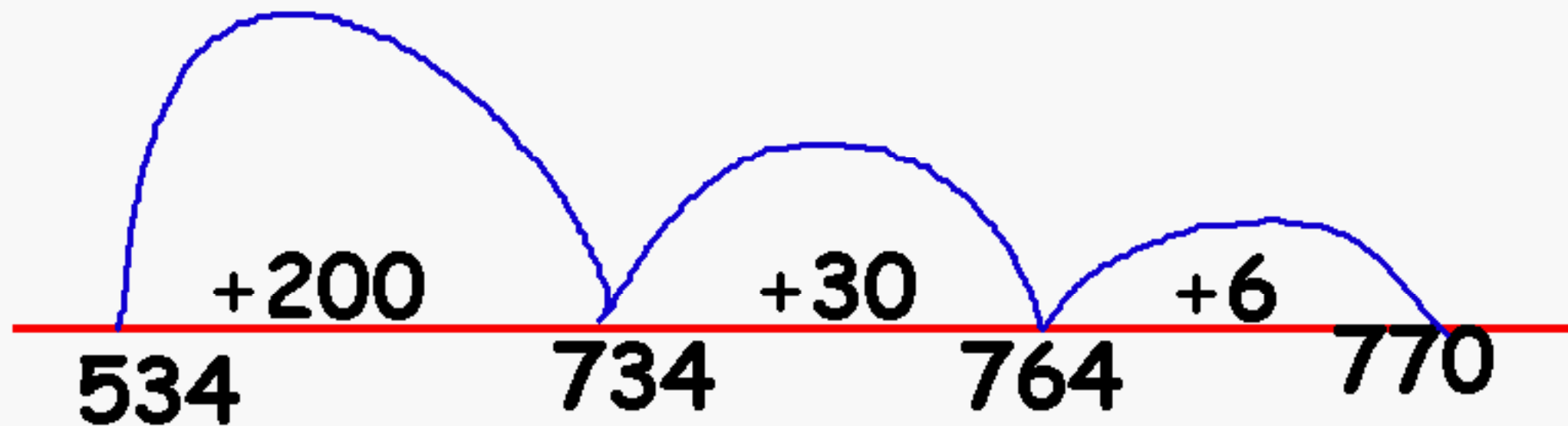


Year 2- The empty number line

Children who are confident to add multiples of 10 will be taught to add 3 digit numbers using an empty number line and partitioning .

$$534 + 236 =$$

$$534 + 200 + 30 + 6 =$$



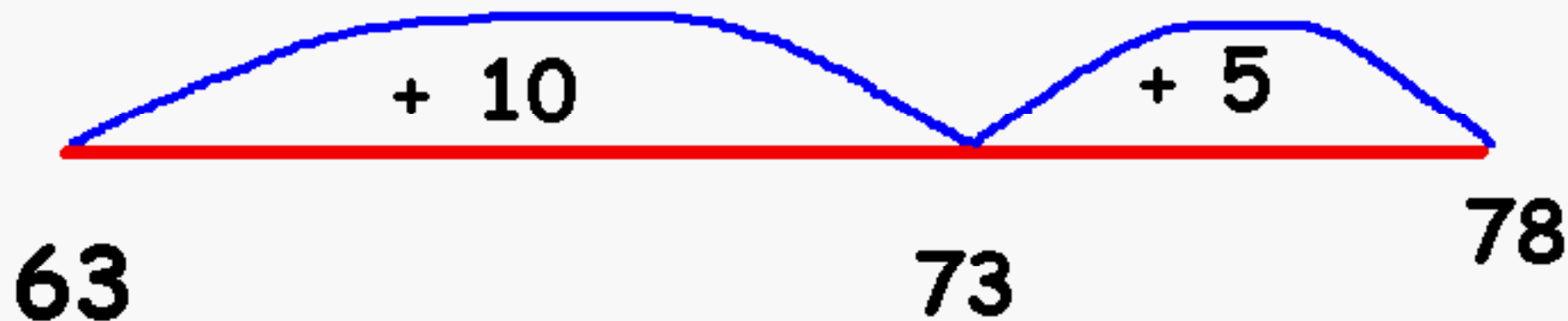
Calculations - Addition

Year 2 - 3 The empty numberline

Children are taught to Partition (separate into tens and units) and start from the left side of the numberline.

$$63 + 15 =$$

$$63 + 10 + 5$$



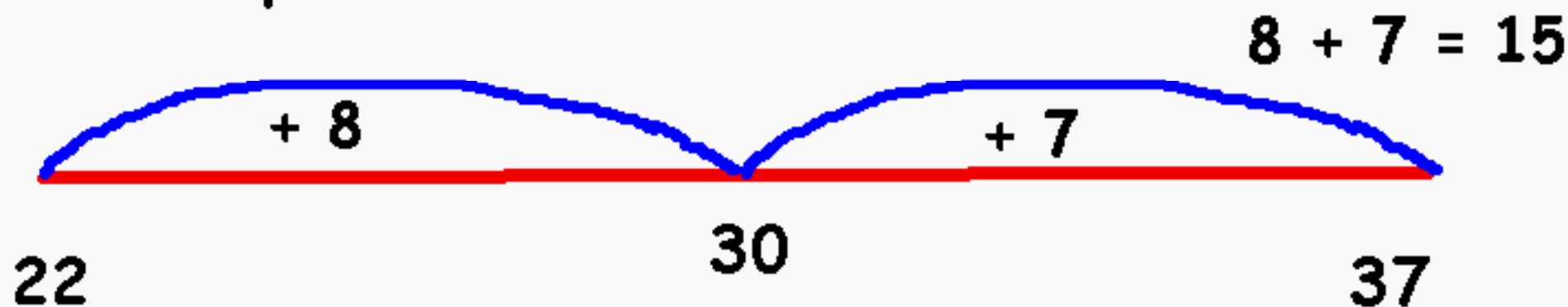
Year 3 The empty numberline 1

10

Children are taught to find the difference between two numbers by counting on (adding). Put the smallest number on the left. Count on to the next ten, write how many you have jumped. Count on to the final number, count how many you have jumped. Add the number of jumps together.

What is the difference between 22 and 37

Jump to next 10 first

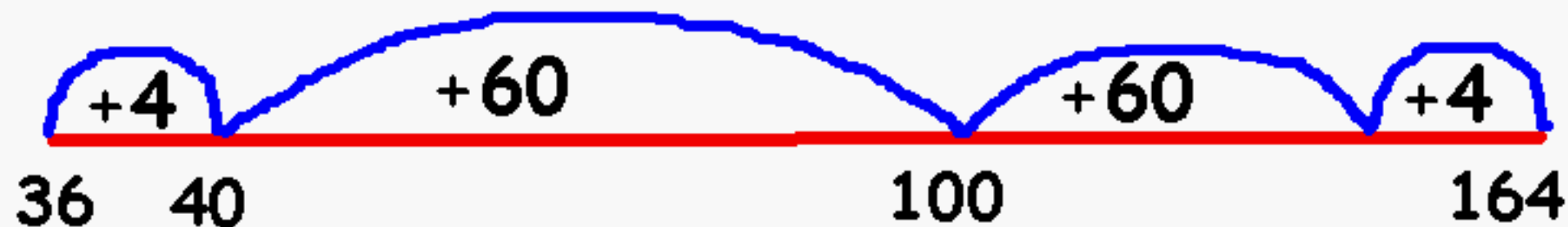


Year 3 The empty numberline 2

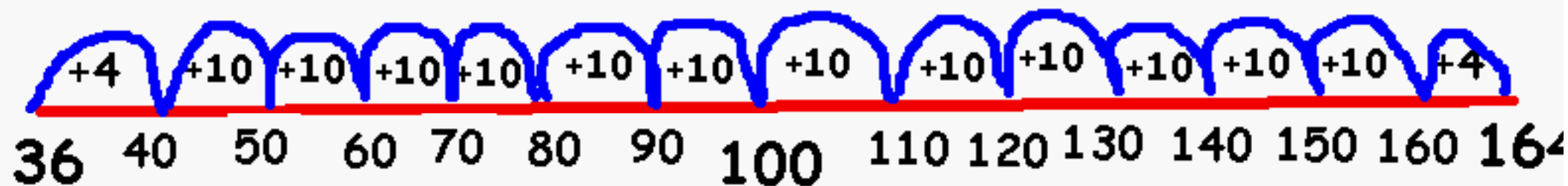
Find the difference between two numbers by counting on (adding)

What is the difference between 36 and 164

Jump to next 10 first - then to next 100



Children may prefer to jump in 10's when less confident with numbers.



Year 3 Partition to add large numbers.

Leading to vertical addition

With larger numbers, using number lines may not be the most efficient method.

First partition into hundreds, tens and units and add separately, starting with the units. Finally recombine or add totals together.

$$324 + 143 =$$

$$\begin{array}{r} 324 \\ + 143 \\ \hline 467 \end{array}$$

300	20	4
100	40	3
400	60	7

$$= 467$$

Year 3 Partition to add large numbers.

Leading to vertical addition

With larger numbers, using numberlines may not be the most efficient method.

First partition into hundreds, tens and units and add separately, starting with the units. Finally add totals together. To add 50 and 15, children may partition again and add 50 + 10 + 5.

528	+	137	+	500	20	8	
665				100	30	7	
665				600	50	15	

Year 4 Partition to add leading to column method

14

As children become more confident with place value (what each digit in a number represents) they are taught to recognise that $8 + 7 = 15$ and that $15 = 10 + 5$, so the 10 needs to be written under the tens column to be added with the other 10's.

$$\begin{array}{r} 528 \\ + 137 \\ \hline \\ \hline \end{array}$$

+

500	20	8
100	30	7
600	50	15

$$\begin{array}{r} \text{HTU} \\ 528 \\ + 137 \\ \hline 665 \\ \hline 1 \end{array}$$

Year 5 and 6 column method

Children use column method to add thousands, hundreds, tens and units. First (a) bridging tens, then (b) bridging hundreds, then (c) bridging thousands then (d) applying place value knowledge to decimals. With decimals remember the numbers move, not the decimal point.

(a)

HTU
528
+137
<hr style="border: 0.5px solid black;"/>
665
<hr style="border: 0.5px solid black;"/>

(b)

HTU
475
+147
<hr style="border: 0.5px solid black;"/>
622
<hr style="border: 0.5px solid black;"/>

(c)

Th H T U
1528
+ 737
<hr style="border: 0.5px solid black;"/>
2265
<hr style="border: 0.5px solid black;"/>

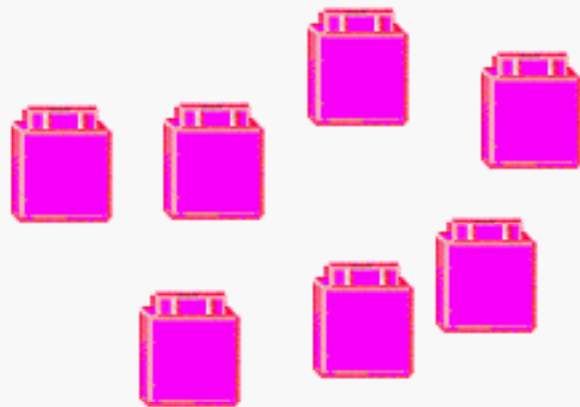
(d)

TU . t h
15 . 28
 7 . 37
<hr style="border: 0.5px solid black;"/>
22 . 65
<hr style="border: 0.5px solid black;"/>

Calculations - Subtraction

Year 1

Children begin by taking away objects from a given set



$$7 - 4 =$$

Subtraction from 10 using fingers



8

-



5

=

Year 1 Subtracting numbers up to 20

18

Start from the largest number on a number strip and then count back.



$$14 - 5 =$$

Year 1 Using a hundred square

19

Subtracting a 1 digit number from a 2 digit number

When working with larger numbers children are taught to start from the largest number on a 100 square and count back.

$$53 - 6 =$$

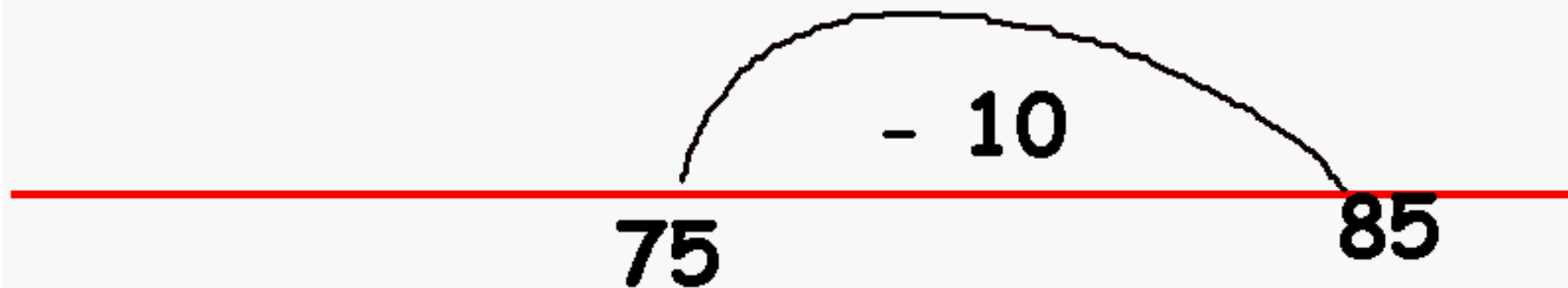
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Year 2 The empty number line

20

The children are taught to subtract 10 using an empty number line. They start by putting the large number on the right hand side of the empty number line and then drawing on the jump of 10.

$$85 - 10 =$$

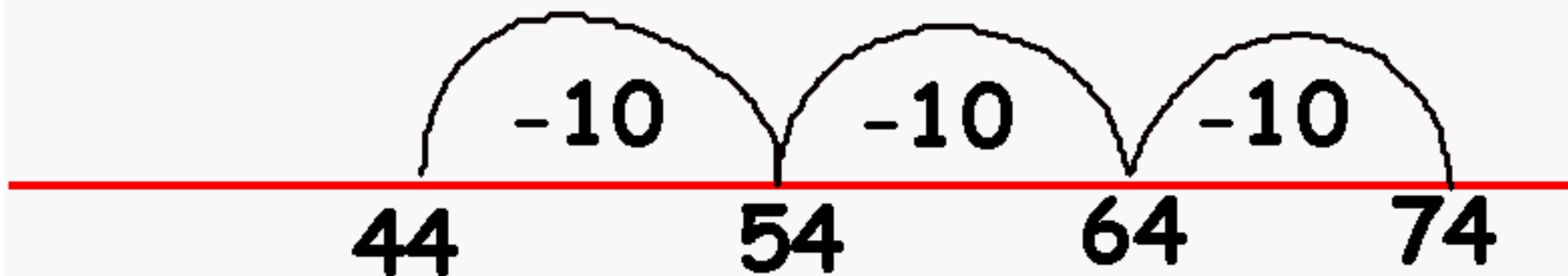


Year 2 The empty number line

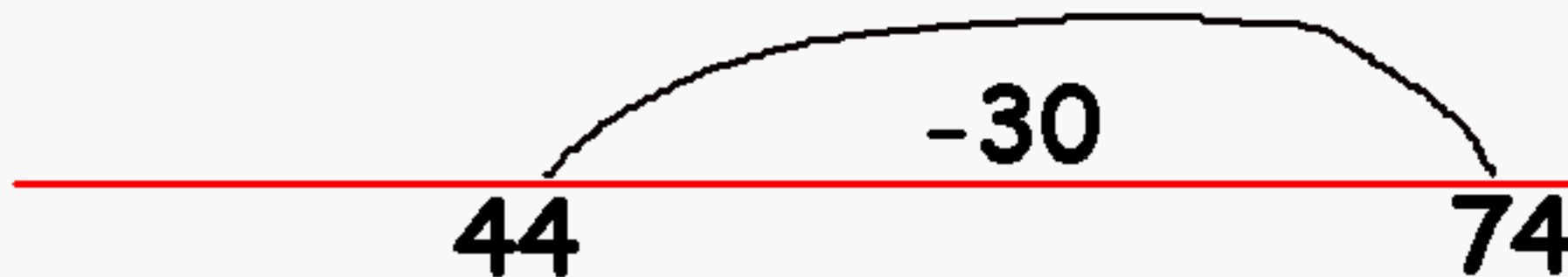
21

Children start on the right hand side of the number line and then subtracting multiples of 10 using an empty number line.

$$74 - 30 =$$



Children confident to subtract multiples of 10 will record this in one jump

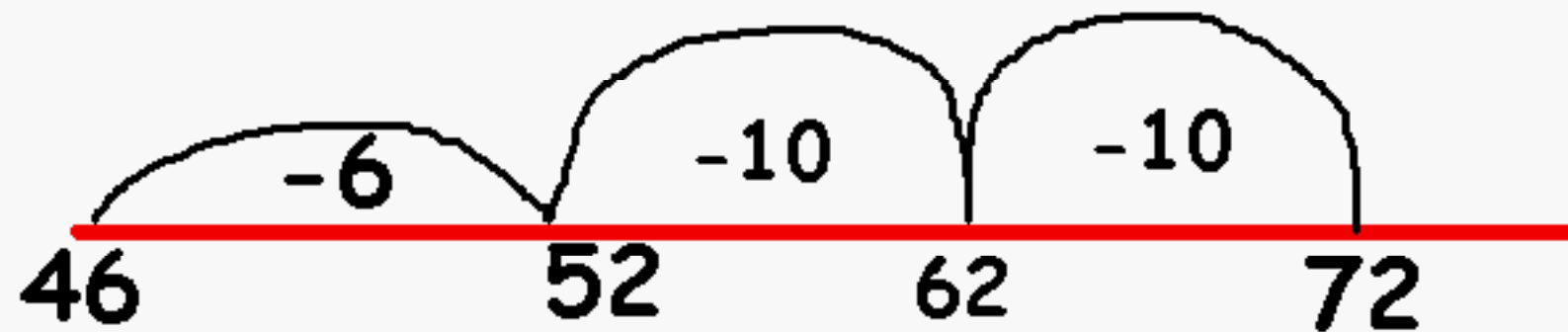


Year 2 The empty number line

When subtracting a 2 digit number using an empty numberline children partition the smaller number into tens and units. Then record on an empty numberline starting on the right hand side.

$$72 - 26 =$$

$$72 - 20 - 6 =$$



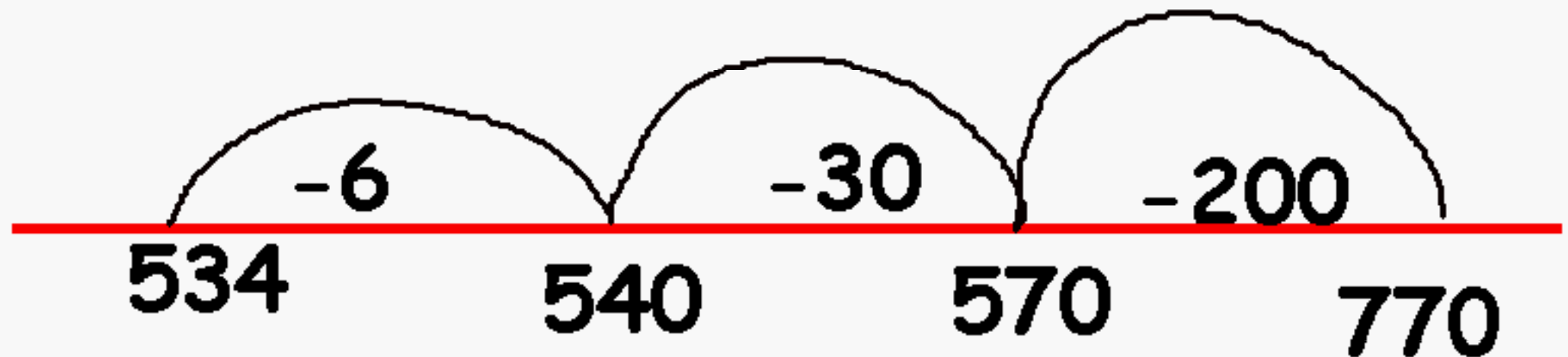
Year 2 The empty number line

23

Children can subtract 3 digit numbers using an empty number line and partitioning .

$$770 - 236 =$$

$$770 - 200 - 30 - 6 =$$



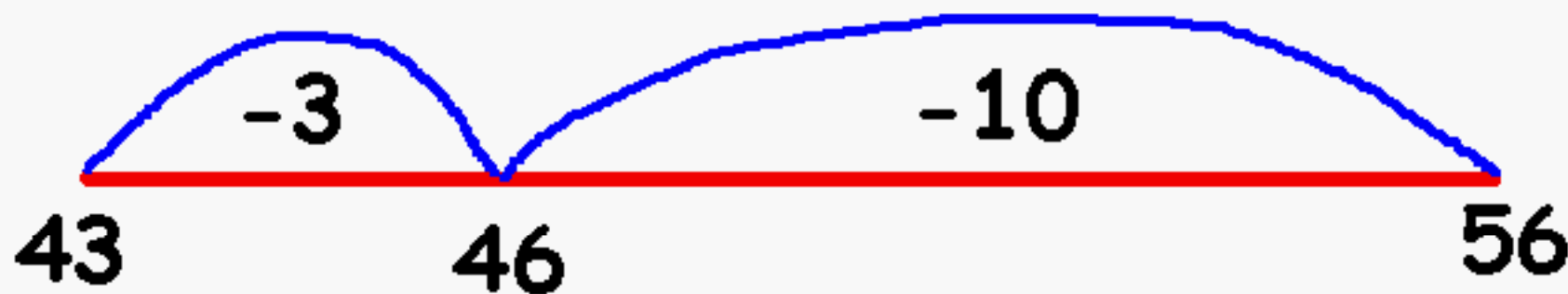
Calculations - Subtraction

Year 2 - 3 The empty numberline

Write the starting number on the right hand side of the number line. Partition the number to be subtracted into tens and units and count back.

$$56 - 13 =$$

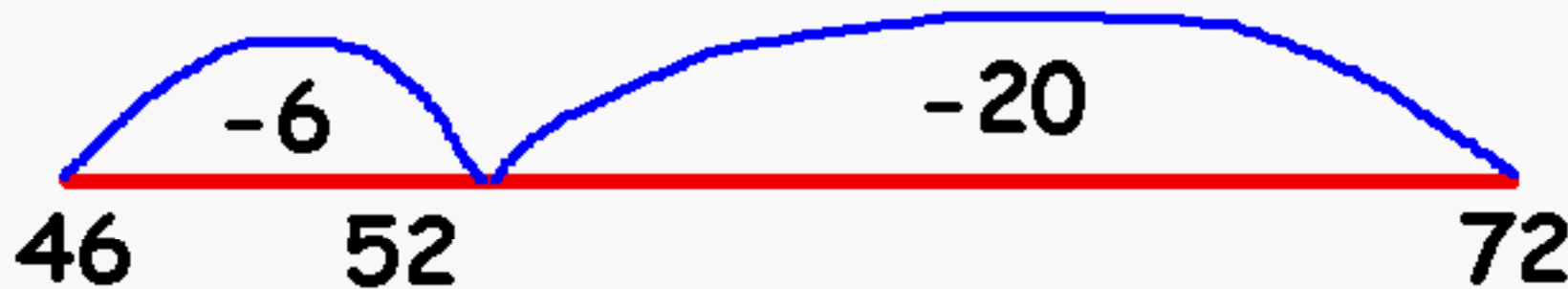
$$56 - 10 - 3$$



Year 3 The empty numberline

Write the starting number on the right hand side of the number line. Partition the number to be subtracted into tens and units and count back. Then recombine the number of jumps.

$$72 - 26 =$$



Partition to subtract large numbers.

Leading to vertical subtraction

With larger numbers, using number lines may not be the most efficient method.

First partition into hundreds, tens and units and subtract separately, starting with the units. Finally recombine numbers together.

$$384 - 143 =$$

$$\begin{array}{r} 384 \\ - 143 \\ \hline 241 \end{array}$$

300	80	4
100	40	3
200	40	1

Partition to subtract - change a ten

Sometimes there is a problem with this method because the units number is less than the number to be subtracted.

In this case we need to change a ten.

Adjust the numbers by taking a ten from the tens column then subtract, starting with the units.

Recombine the tens and units to find the answer.

$$\begin{array}{r} 60 \\ -36 \\ \hline 24 \end{array}$$

60	0
30	6

Partitioned numbers

50	10
30	6
20	4

Adjusted numbers

Year 3 Partition to subtract change a ten

Sometimes there is a problem with this method because the units number is less than the number to be subtracted.

In this case we need to change a ten.

Adjust the numbers by taking a ten from the tens column.

Children add this to the number in the units column.

Then subtract, starting with the units.

Recombine the tens and units to find the answer.

$$\begin{array}{r} 62 \\ -48 \\ \hline 14 \end{array}$$

60	2
40	8

Partitioned numbers

50	12
40	8
10	4

Adjusted numbers

If there is problem in the tens column - the number to be subtracted is more than the starting number -you need to change a hundred.
Adjust the hundreds and tens so the subtraction is possible.

$$\begin{array}{r}
 728 \\
 - 263 \\
 \hline
 465
 \end{array}$$

700	20	8
200	60	3

Partitioned numbers

600	120	8
200	60	3
400	60	5

Adjusted numbers

Partition to subtract change a ten and hundred

Sometimes there may be lower numbers in both the units and the tens columns. in this case, first adjust to change the tens, then the hundreds.

$$\begin{array}{r} 721 \\ - 263 \\ \hline 458 \end{array}$$

700	20	1
200	60	3

Partitioned numbers

	110	11
600	120	1
200	60	3
400	50	8

Adjusted numbers

When children are secure in their understanding of place value (what each digit in a number represents) the partitioning method can be replaced by the shorthand column method. This is where you indicate changing tens and hundreds as in the method shown below - decomposition. This would be introduced in steps - firstly by changing tens...

		40	16									
456	<table border="1"><tr><td>400</td><td>50</td><td>6</td></tr><tr><td>100</td><td>30</td><td>8</td></tr><tr><td>300</td><td>10</td><td>8</td></tr></table>	400	50	6	100	30	8	300	10	8		
400	50	6										
100	30	8										
300	10	8										
-138				<table border="0"><tr><td>4</td><td>1</td></tr><tr><td>456</td><td></td></tr><tr><td>-138</td><td></td></tr><tr><td><u>318</u></td><td></td></tr></table>	4	1	456		-138		<u>318</u>	
4	1											
456												
-138												
<u>318</u>												
<u> </u>												

...then hundreds....

418	300	110	
<u>-133</u>	400	10	8
—	100	30	3
	200	80	5

³~~**418**~~
-133
285

... then both tens and hundreds.

		100	
	300	110	18
418	400	10	8
<u>-139</u>	100	30	9
—	200	70	9

3	10	18
4	1	8
<u>-139</u>		
<u>279</u>		

Year 5 and 6 column method - Decomposition

Decomposition - this means adjusting numbers so that subtraction is possible.

Children use column method to subtract numbers in thousands, hundreds, tens and units. First (a) no decomposition needed (b) decomposing tens, then (c) decomposing hundreds, then (d) decomposing thousands then applying place value knowledge to decomposing decimals.

$$\begin{array}{r} \text{(a)} \\ \text{HTU} \\ 548 \\ - 137 \\ \hline 411 \end{array}$$

$$\begin{array}{r} \text{(b)} \\ \text{HTU} \\ \quad 6 \ 15 \\ 475 \\ - 147 \\ \hline 325 \end{array}$$

$$\begin{array}{r} \text{(c)} \\ \text{Th H T U} \\ \quad 4 \ 12 \\ 1528 \\ - 237 \\ \hline 1291 \end{array}$$

$$\begin{array}{r} \text{(d)} \\ \quad 2 \ 15 \\ 3548 \\ - 737 \\ \hline 2811 \end{array}$$

Year 5 and 6 column method - Decomposition

Decomposition - this means adjusting numbers so that subtraction is possible.

Children use column method to subtract numbers in thousands, hundreds, tens and units.

Decomposing hundreds into tens and units.

By using this method
the children can see that
the value is still 400
 $300 + 90 + 10 = 400$

(a)
HTU

$$\begin{array}{r} 400 \\ - 147 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} 3 \ 10 \\ \cancel{400} \\ - 147 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} 3 \ 9 \ 10 \\ \cancel{400} \\ - 147 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} 3 \ 9 \ 10 \\ \cancel{400} \\ - 147 \\ \hline 253 \\ \hline \end{array}$$

Calculations Multiplication

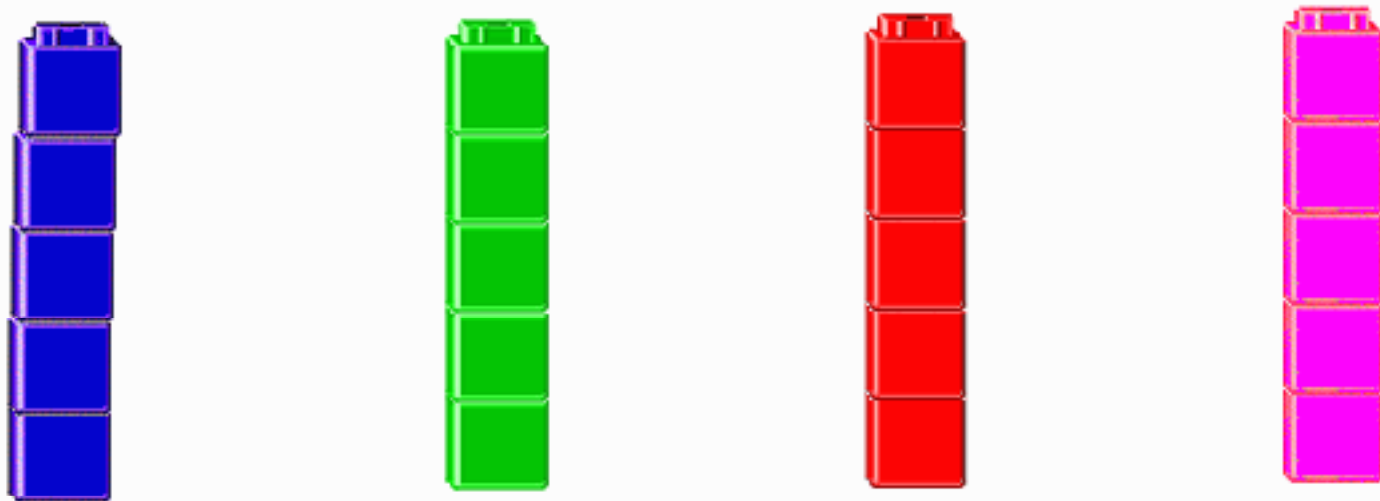
Year 1

Children practice counting socks and fingers etc in 2's, 10's and 5's as appropriate



Year 1 Making equal groups

Children use objects to make equal groups



4 lots of 5

Year 2 Array

Children draw an array

$$4 \times 3 =$$

$$4 \text{ lots of } 3 =$$

1 lot of 3	x	x	x	3
2 lots of 3	x	x	x	6
3 lots of 3	x	x	x	9
4 lots of 3	x	x	x	12

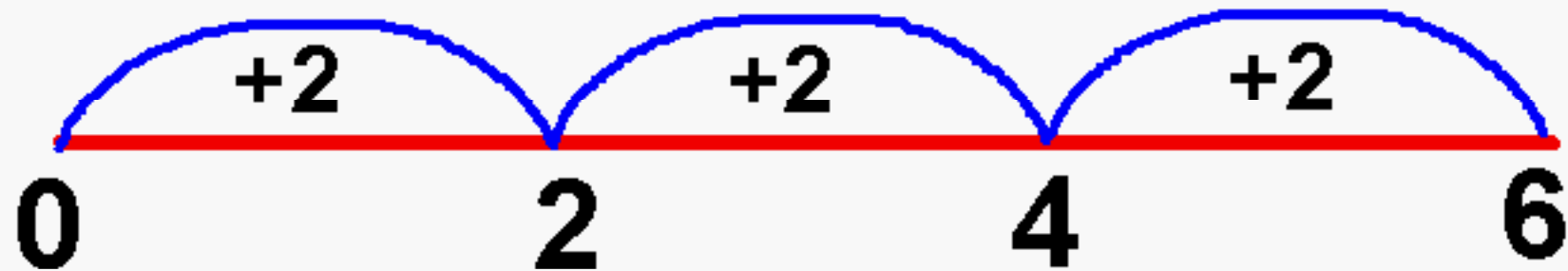
Year 2 The empty number line

Children are taught to show 3 groups of 2, as 3 jumps of 2 on the number line, then add the total number of jumps.

$$3 \times 2 =$$

3 lots of 2

As repeated addition
- Jumps of the same amount
on a numberline



Year 2 Partition to multiply

Children are taught to multiply a 2 digit numbers by a single digit by partitioning the largest number.

$$13 \times 5 =$$

The diagram illustrates the partitioning of the number 13 into 10 and 3. Two blue lines originate from the '1' and '3' in the equation $13 \times 5 =$ and point down to the '10' and '3' in the equation $10 \times 5 + 3 \times 5 =$ respectively. Below this, the result of each multiplication is shown: $50 + 15 =$.

$$13 \times 5 =$$
$$10 \times 5 + 3 \times 5 =$$
$$50 + 15 =$$

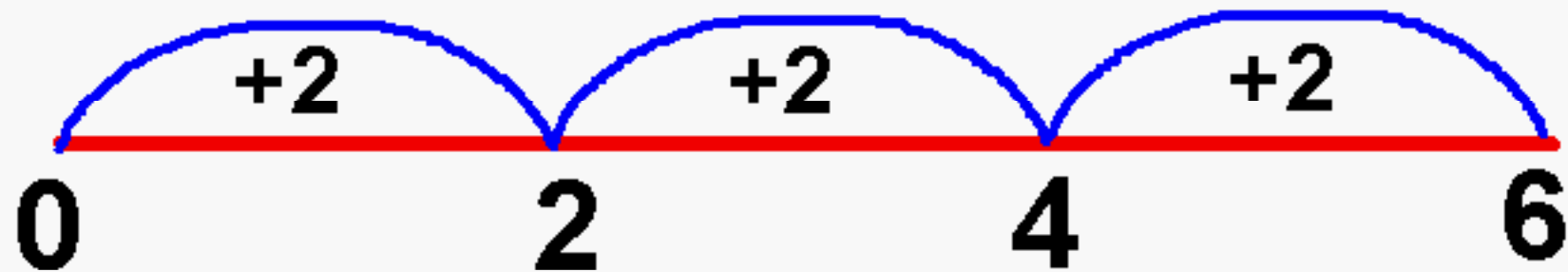
Calculations - Multiplication

Year 2 - 3 The empty numberline

Children are taught to show 3 groups of 2 as 3 jumps of 2 on number line, then asked to add the total number of jumps.

$$3 \times 2 =$$

As repeated addition
- Jumps of the same amount
on a numberline



Year 2 - 3 As arrays

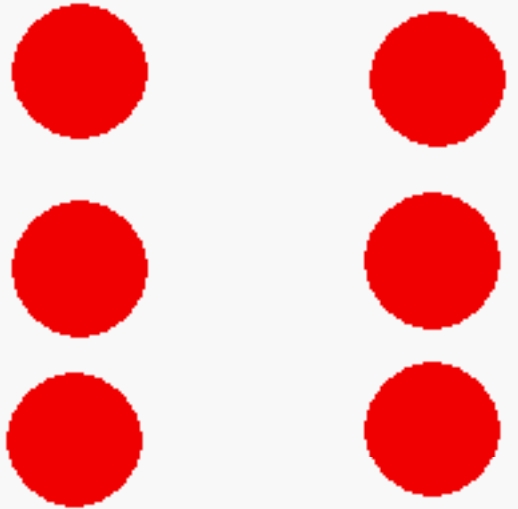
Children find out 3×2 by drawing arrays.

$$3 \times 2 = \begin{array}{c} \bullet \\ \bullet \\ 2 \end{array} \quad \begin{array}{c} \bullet \\ \bullet \\ + 2 \end{array} \quad \begin{array}{c} \bullet \\ \bullet \\ + 2 \end{array}$$

3 groups of 2 = 6

Year 2 - 3 As arrays

Children find out 2×3 by drawing arrays.

$$2 \times 3 =$$


3 **+ 3**

2 groups of 3 = 6

Year 3 - Partition to multiply

With larger numbers - partition into tens and units and multiply separately. Then recombine or add the totals.

$$\begin{array}{r} 33 \\ \times 2 \\ \hline 66 \end{array}$$

$$\begin{array}{r} \mathbf{X} \\ \mathbf{2} \end{array} \begin{array}{|c|c|} \hline \mathbf{30} & \mathbf{3} \\ \hline \mathbf{60} & \mathbf{6} \\ \hline \end{array} = \mathbf{66}$$

Year 3 - Partition to multiply

45

With larger numbers - partition into hundred, tens and units and multiply separately. Then add the totals.

$$\begin{array}{r} 361 \\ \times \quad 6 \\ \hline 1986 \end{array}$$

X
6

300	60	1
1800	180	6

=1986

Year 3 - Multiply by 10

X 10

$$4 \times 10 = 40$$

Th	H	T	U
0	0	0	4

Th	H	T	U
0	0	4	0



To multiply by 10, the number moves 1 place to the left and is worth ten times more.

X 10

$$14 \times 10 = 140$$

Th	H	T	U
0	0	1	4

Th	H	T	U
0	1	4	0



To multiply 2 digit numbers by 10, both numbers move 1 place to the left and are worth ten times more.

To understand place value.
Place value - the value of the
number each digit represents

Year 3 - Multiply by 10 To understand place value.47

X 100

$$4 \times 100 = 400$$

Th	H	T	U
0	0	0	4

Th	H	T	U
0	4	0	0



To multiply by 100, the number moves 2 places to the left and is worth a hundred times more.

X 100

$$14 \times 100 = 1400$$

Th	H	T	U
0	0	1	4

Th	H	T	U
1	4	0	0



To multiply 2 digit numbers by 100, both numbers move 2 places to the left and are worth a hundred times more.

Year 4 Partition to multiply leading to short multiplication

48

When children are secure in their understanding of place value (what each digit in a number represents) the partitioning method can be replaced by the shorthand column method. $6 \times 4 = 24$

$24 = 20 + 4$ so the 20 can be written as 2 tens under the tens column.

Then add the totals together.

$\begin{array}{r} 314 \\ \times 6 \\ \hline \hline \end{array}$	X	<table border="1"><tr><td>300</td><td>10</td><td>4</td></tr><tr><td>1800</td><td>60</td><td>24</td></tr></table>	300	10	4	1800	60	24	$\begin{array}{r} 314 \\ \times 6 \\ \hline 1884 \\ 2 \end{array}$
300	10	4							
1800	60	24							

Year 5 Grid method (cabbage patch method)

$$\begin{array}{r} 549 \\ \times 6 \\ \hline 3294 \end{array}$$

Partition into hundreds tens and units.
Then multiply separately.
Finally add the totals together.

X	500	40	9
6	3000	240	54

Add either horizontally
 $3000 + 240 + 54 = 3294$

or vertically

$$\begin{array}{r} 3000 \\ 240 \\ + 54 \\ \hline 3294 \end{array}$$

Year 5 Grid method (cabbage patch method)

$$\begin{array}{r} 549 \\ \times 26 \\ \hline 14274 \end{array}$$

Multiply by 2 digits.

Partition into hundreds tens and units.

Then multiply separately.

Finally add the totals together.

X	500	40	9
6	3000	240	54
20	10000	800	180

Add either horizontally

$$3000 + 240 + 54 = 3294$$

$$10000 + 800 + 180 = \underline{10980} +$$

$$\underline{14274}$$

or vertically

$$\begin{array}{r} 3000 \\ 240 \\ + 54 \\ \hline 3294 \end{array}$$

$$\begin{array}{r} 10000 \\ 800 \\ + 180 \\ \hline 10980 \end{array}$$

$$\underline{3294}$$

$$\underline{10980}$$

$$+ \underline{10980}$$

$$\underline{14274}$$

Year 5 and Year 6 standard vertical multiplication by partitioning.

$$\begin{array}{r} 36 \\ \times 18 \\ \hline 648 \end{array}$$

$$\begin{array}{r} 36 \\ \times 10 \\ \hline 360 \end{array}$$

$$\begin{array}{r} 36 \\ \times 8 \\ \hline 288 \end{array}$$

$$\begin{array}{r} 360 \\ +288 \\ \hline 648 \end{array}$$

Multiply by 2 digits.

Partition into tens and units.

Then multiply separately.

Finally add the totals together.

Year 5 and Year 6 standard vertical multiplication.

$$\begin{array}{r} 36 \\ \times 18 \\ \hline 360 \\ 288 \\ 4 \\ \hline 648 \end{array}$$

(Multiply by 10)

(Multiply by 8) Remember to add numbers into their appropriate place value columns.

This method can be applied to hundreds, thousands and decimals.

Calculations

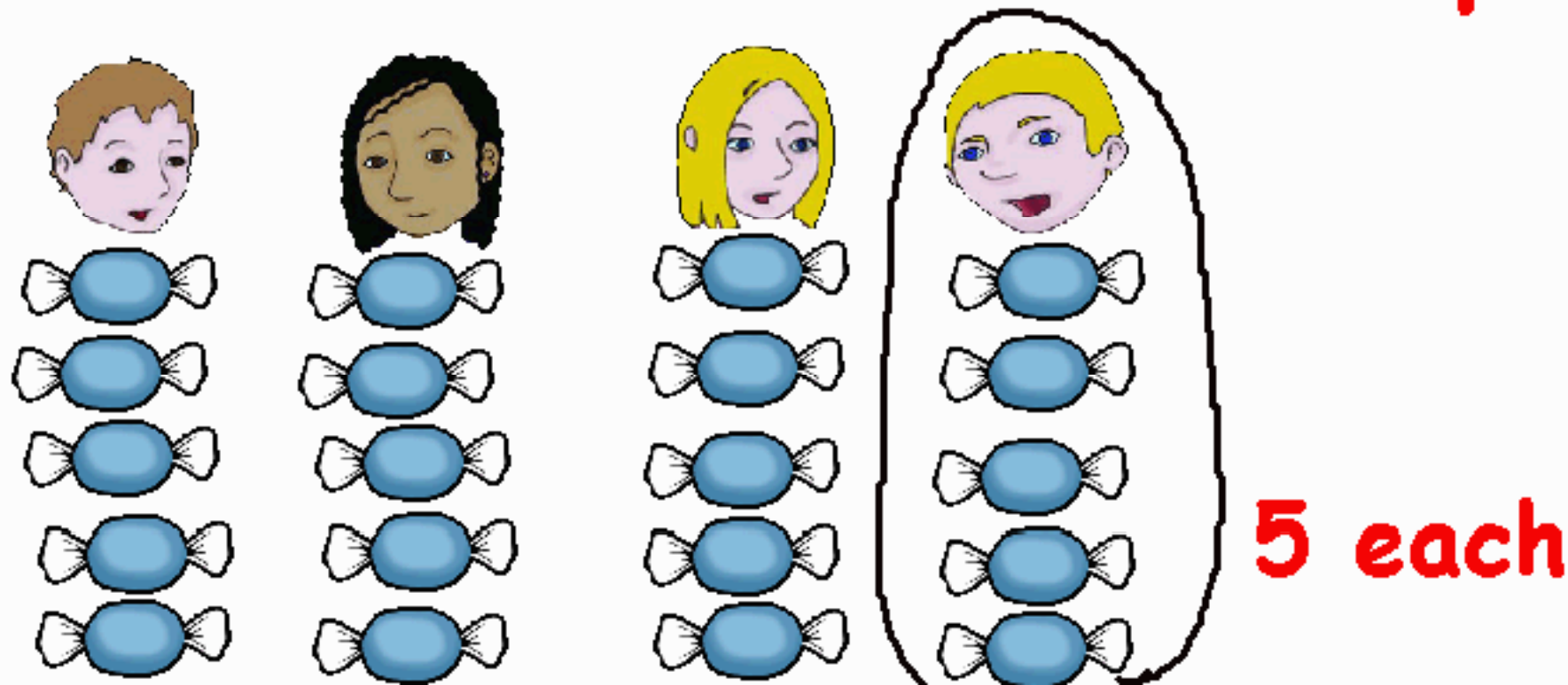
Division

Year 1 and year 2

Children share practically into equal groups, and record their sharing as an array.

$$20 \div 4 =$$

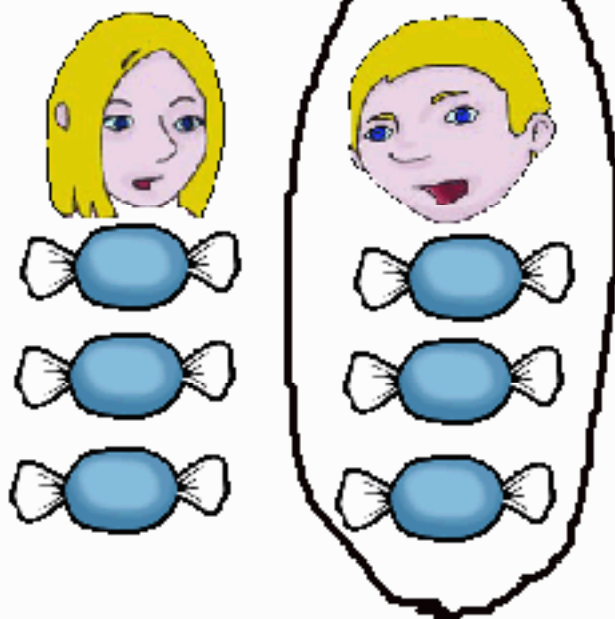
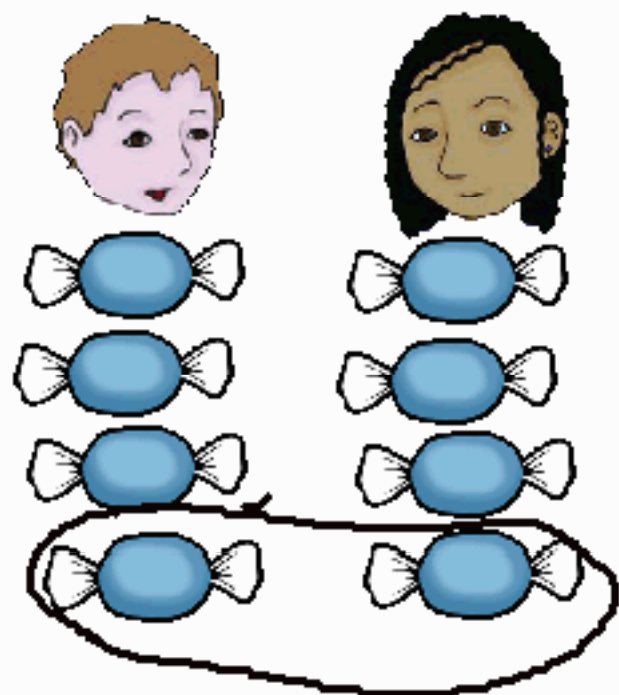
20 shared between 4 equals



Year 2 Sharing into equal groups as an array

Children draw arrays to record their sharing into equal groups with remainders.

$14 \div 4 =$
14 shared between 4 equals



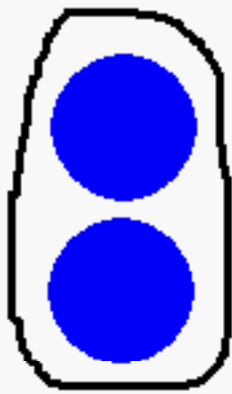
3 each
remainder 2
 $3r2$

Year 2 Dividing into groups ~ arrays

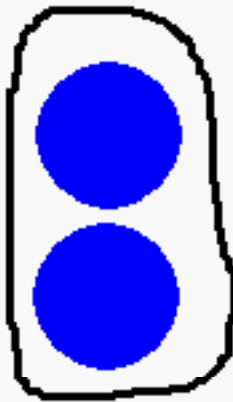
54

Children organise counters into equal groups, and then count the groups. They progress to drawing their groups as arrays

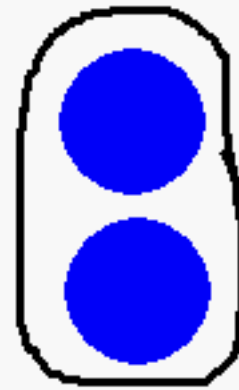
$$6 \div 2 =$$



**1 group
of 2**



**2 groups
of 2**



**3 groups
of 2**

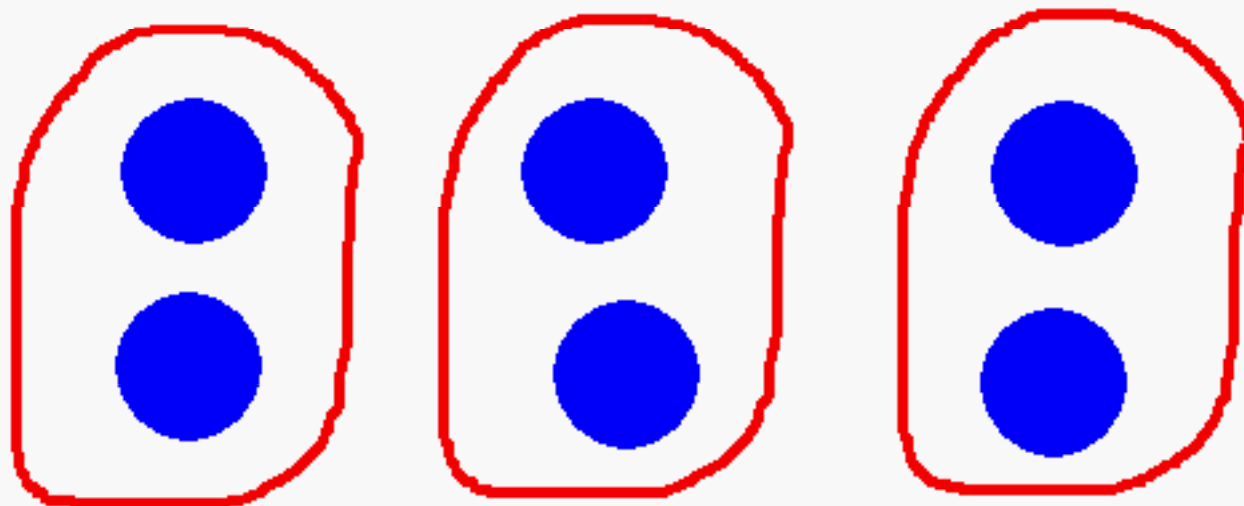
Calculations - Division

Year 2 - 3 Dividing equally into groups - arrays

Children organise counters into groups and count the number of groups.

They progress to drawing arrays -
6 grouped in 2's = 3 groups.

$$6 \div 2 = 3$$



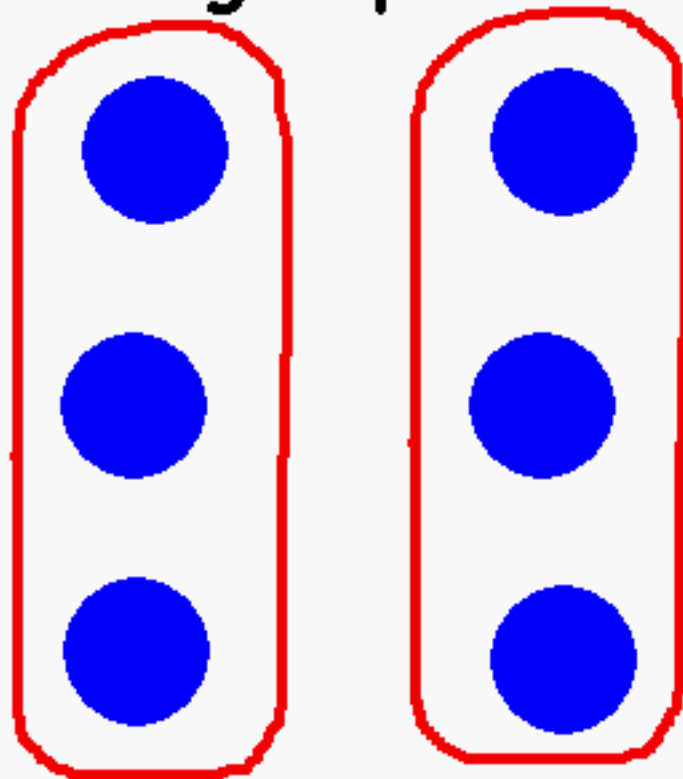
Calculations - Division

Year 2 - 3 Dividing equally into groups - arrays

Children organise counters into groups and count the number of groups.

They progress to drawing arrays -
6 grouped in 3's = 2 groups.

$$6 \div 3 = 2$$



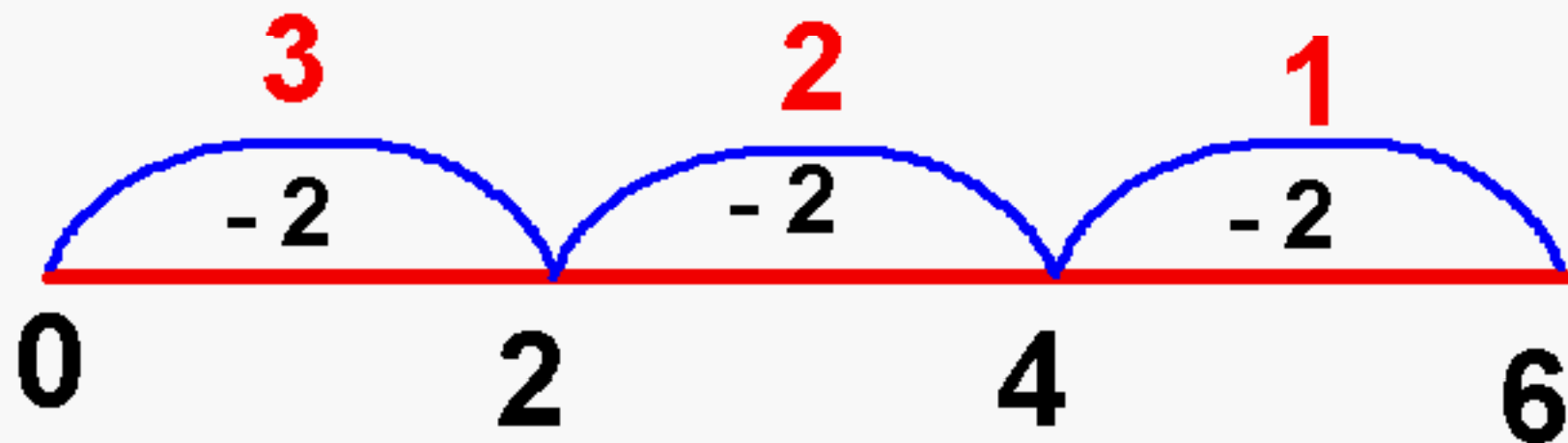
Year 2 - 3 The empty numberline

Children link division to repeated subtraction and show the jumps on a number line.

Starting from the right hand side.

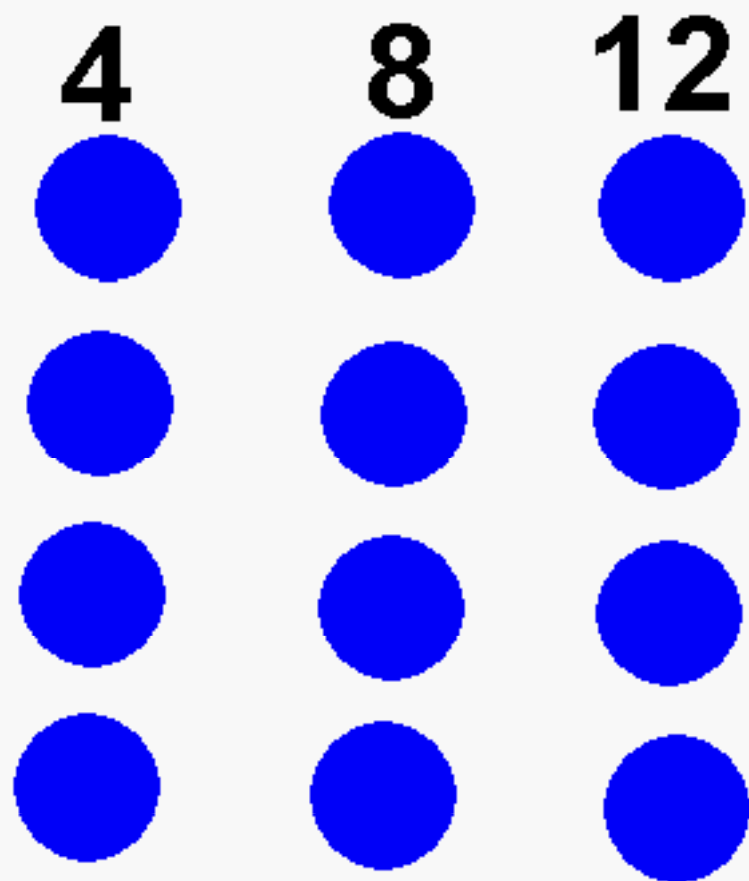
$$6 \div 2 = 3$$

As repeated subtraction
- Jumps of the same amount
on a numberline
How many jumps?



Year 3 Dividing equally into groups and arrays, using Table facts

Children use their knowledge of times tables (which should include quick recall) to work out division facts. Multiplication and division are taught alongside each other as inverse (opposite) operations.



$$12 \div 4 =$$

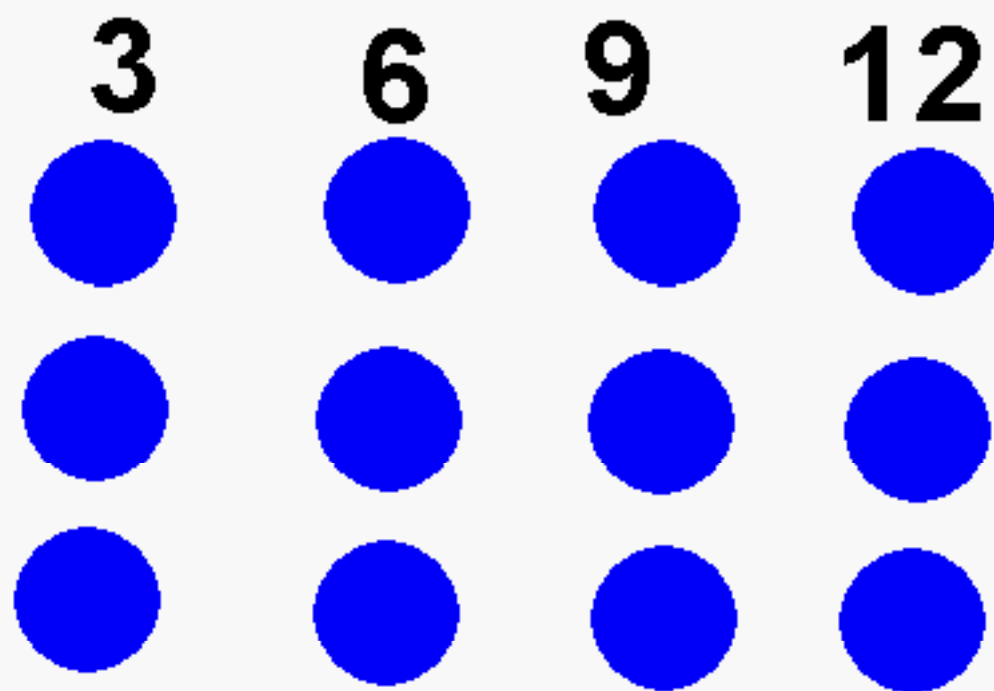
$$12 \div 4 = 3$$

because

$$3 \times 4 = 12$$

Year 3 Dividing equally into groups and arrays, using Table facts

Children use their knowledge of times tables (which should include quick recall) to work out division facts. Multiplication and division are taught alongside each other as inverse (opposite) operations.



$$12 \div 3 =$$

$$12 \div 3 = 4$$

because

$$4 \times 3 = 12$$

Year 3 and year 4

Dividing larger 2 digit numbers by using table facts.

$$48 \div 6 =$$

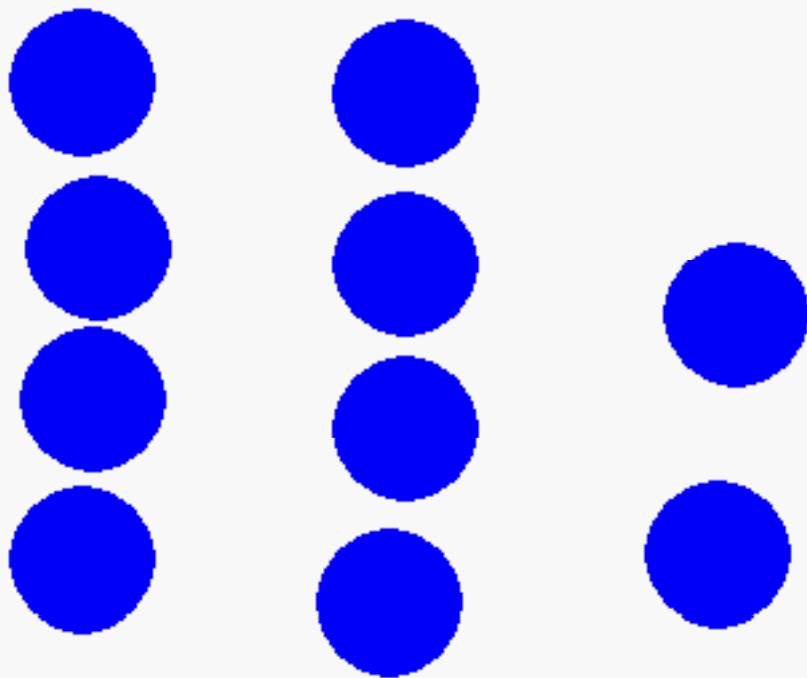
$$48 \div 6 = 8$$

because

$$8 \times 6 = 48$$

Year 3 and four
Dividing with remainders.

$$10 \div 4 =$$



$$10 \div 4 = 2r.2$$

because

$$2 \times 4 = 8$$

+ 2 more to make 10

Years 4, 5 and 6

Dividing larger 2 digit numbers

Short division

$$81 \div 3 = 27$$

$$\begin{array}{r} 27 \\ 3 \overline{) 81} \end{array}$$

Divide the 8 digit (representing 80)
by 3 ($8 \div 3 = 2r.2$)

Write the 2 (representing 20)
next to the 1 to make 21.

Divide 21 by 3 = 7

Answers can be checked by doing the inverse

$$3 \times 27 = 81$$

Years 5 and 6

Dividing larger 3 digit numbers

Short division.

$$292 \div 3 = 97 \text{ r. } 1$$

$$\begin{array}{r} 0 \ 9 \ 7 \ \text{r. } 1 \\ 3 \overline{) 2 \ 2 \ 9 \ 2} \end{array}$$

Divide the 2 digit

(representing 200)

by 3 ($2 \div 3 = 0 \text{ r. } 2$)

Write the 2 (representing 200)
next to the 9 to make 29 tens.

Divide 29 tens by 3 = 7 r.2

Write the 2 (representing 2 tens)
next to the 2 to make 22.

Divide 22 by 3 = 7 r. 1

Answers can be checked by doing the inverse

$$3 \times 97 = 291 + 1(\text{ the remainder}) = 292$$

Years 5 and 6

64

Dividing 3 digit numbers by 2 digit numbers.

Long division

$$560 \div 24 = 23 \text{ r.} 8$$

$$\begin{array}{r} 0 \ 2 \ 3 \ \text{r.} 8 \\ 24 \overline{) 5 \ 6 \ 0} \\ \underline{- 4 \ 8 \ 0} \\ 8 \ 0 \\ \underline{- 7 \ 2} \\ 8 \end{array}$$

Divide the 5 digit

(representing 500)

by 24 ($5 \div 24 = 0$)

Write the 5 (representing 500)

next to the 6 to make 56 tens.

Divide 56 tens by 24 = 2 r.8

Write the 8 (representing 8 tens

next to the 0 to make 80.

Divide 80 by 24 = 3 r.8

Answers can be checked by doing the inverse

$$24 \times 23 = 552 + 8(\text{ the remainder}) = 560$$